L1 - Progress

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If $\Gamma \vdash e : \tau$ and $dom(\Gamma) \subseteq dom(s)$ then either e is a value or $\exists e', s'. \langle e, s\rangle \to \langle e', s'\rangle$

$\mathbb{D}(\Gamma, e, \tau) = \forall s. dom(\Gamma) \subseteq dom(s).$
$value(e) \vee \exists e', s'. \langle e, s\rangle \to \langle e', s'\rangle$

Show that if $\Gamma \vdash e : \tau$ then $\mathbb{D}(\Gamma, e, \tau)$
by induction over the derivation of $\vdash$

Case (int), (bool), (skip)
$e = n$, true, false or skip — value(e) holds

Case (op+)
$e = e_1 + e_2$ and $\Gamma \vdash e_1 : int$, $\Gamma \vdash e_2 : int$
and by induction $\mathbb{D}(\Gamma, e_1, int)$ ①, $\mathbb{D}(\Gamma, e_2, int)$ ②
show $(value(e_1 + e_2) \vee \exists e', s'. \langle e_1 + e_2, s\rangle \to \langle e', s'\rangle$
need to show the latter!
Case ① $\langle e_1, s\rangle \to \langle e_1', s'\rangle$
Then $\langle e_1 + e_2, s\rangle \to \langle e_1' + e_2, s'\rangle$ using (op1) — done
Case ① value(e_1) $e_1 = n_1$ by INVERSION
Case ② $\langle e_2, s\rangle \to \langle e_2', s'\rangle$
Then $\langle n_1 + e_2, s\rangle \to \langle n_1 + e_2', s'\rangle$ using (op2) — done
Case ② value(e_2) $e_2 = n_2$ by INVERSION
Then $\langle n_1 + n_2, s\rangle \to \langle n, s\rangle$ (where $n = n_1 + n_2$)

Case (deref)
$e = !l$, have $l \in dom(\Gamma)$
show value(!l) $\vee$ $(\exists e', s'. \langle !l, s\rangle \to \langle e', s'\rangle)$
need to show the latter
since $dom(\Gamma) \subseteq dom(s)$, $l \in dom(s)$, $s(l) = n$
and $\langle !l, s\rangle \to \langle n, s\rangle$ using (deref)

Remaining cases
Similarly, see Cambridge notes

INVERSION

If $\Gamma \vdash e : \tau$ and value(e) and
$\tau = int$ then $e = n$
$\tau = unit$ then $e = skip$
$\tau = bool$ then $e = true \vee e = false$

Proof by examination / "induction" over $\vdash$